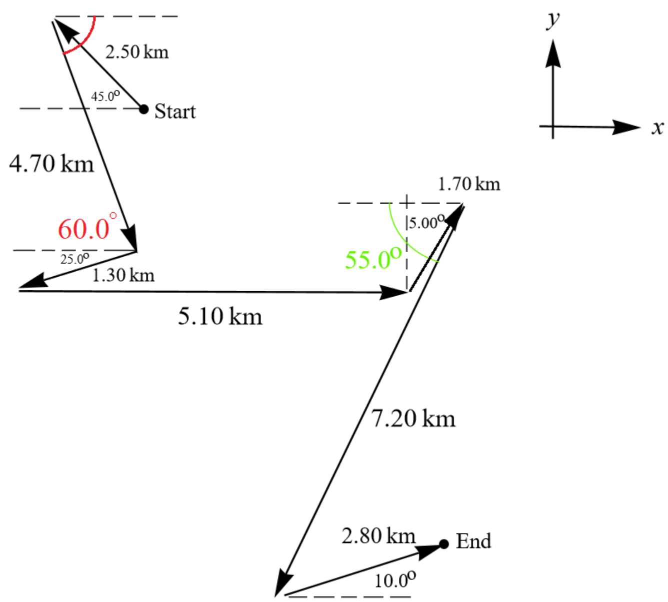
Problem 51

In an attempt to escape a desert island, a castaway builds a raft and sets out to sea. The wind shifts a great deal during the day, and she is blown along the following straight lines: 2.50 km and 45.0° north of west, then 4.70 km and 60.0° south of east, then 1.30 km and 25.0° south of west, then 5.10 km due east, then 1.70 km and 5.00° east of north, then 7.20 km and 55.0° south of west, and finally 2.80 km and 10.0° north of east. Use the analytical method to find the resultant vector of all her displacement vectors. What is its magnitude and direction?

Solution

Let the positive y-axis point north, and let the positive x-axis point east. Draw a schematic of the girl's path.



Find the resultant of all the displacement vectors. Note that when an angle above or below a horizontal is given, multiplying the magnitude by the cosine of the angle gives the horizontal component, and multiplying the magnitude by the sine of the angle gives the vertical component. On the other hand, when an angle to the left or right of a vertical is given, multiplying the magnitude by the sine of the angle gives the horizontal component, and multiplying the magnitude by the cosine of the angle gives the vertical component.

$$\begin{aligned}
\mathbf{d} &= \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \mathbf{d}_4 + \mathbf{d}_5 + \mathbf{d}_6 + \mathbf{d}_7 \\
&= (-2.50\cos 45.0^\circ\,\hat{\mathbf{x}} + 2.50\sin 45.0^\circ\,\hat{\mathbf{y}})\text{ km} + (4.70\cos 60.0^\circ\,\hat{\mathbf{x}} - 4.70\sin 60.0^\circ\,\hat{\mathbf{y}})\text{ km} \\
&\quad + (-1.30\cos 25.0^\circ\,\hat{\mathbf{x}} - 1.30\sin 25.0^\circ\,\hat{\mathbf{y}})\text{ km} + (5.10\hat{\mathbf{x}} + 0\hat{\mathbf{y}})\text{ km} \\
&\quad + (1.70\sin 5.00^\circ\,\hat{\mathbf{x}} + 1.70\cos 5.00^\circ\,\hat{\mathbf{y}})\text{ km} + (-7.20\cos 55.0^\circ\,\hat{\mathbf{x}} - 7.20\sin 55.0^\circ\,\hat{\mathbf{y}})\text{ km} \\
&\quad + (2.80\cos 10.0^\circ\,\hat{\mathbf{x}} + 2.80\sin 10.0^\circ\,\hat{\mathbf{y}})\text{ km}
\end{aligned}$$

Combine like-terms.

$$\begin{aligned}\mathbf{d} &= (-2.50\cos 45.0^\circ + 4.70\cos 60.0^\circ - 1.30\cos 25.0^\circ + 5.10 + 1.70\sin 5.00^\circ - 7.20\cos 55.0^\circ + 2.80\cos 10.0^\circ)\,\mathrm{km}\,\hat{\mathbf{x}} \\ &\quad + (2.50\sin 45.0^\circ - 4.70\sin 60.0^\circ - 1.30\sin 25.0^\circ + 1.70\cos 5.00^\circ - 7.20\sin 55.0^\circ + 2.80\sin 10.0^\circ)\,\mathrm{km}\,\hat{\mathbf{y}} \\ &\approx (3.21\text{ km})\hat{\mathbf{x}} - (6.57\text{ km})\hat{\mathbf{y}}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{d}| &\approx \sqrt{(3.21\text{ km})^2 + (-6.57\text{ km})^2} \\ &\approx 7.31\text{ km},\end{aligned}$$

and its direction is

$$\theta \approx \tan^{-1}\left(\frac{-6.57}{3.21}\right) \approx -64.0^\circ$$

about 64.0° south of east. The resultant is the vector pointing from the start to the end.